WAVEFORM TOMOGRAPHY - SUCCESSES, CAUTIONARY TALES, AND FUTURE DIRECTIONS

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Major objectives

I would like to highlight some of the issues in evaluating the success (or otherwise) of waveform tomography. For synthetic data it is always tempting to commit inverse crimes, for real data a scrutiny of the detailed data fit is "sine qua non". I'll go through some new animations that illustrate the benefits of the frequency domain, and I'll finish with some topics that we are working on - real data examples of Q-inversion being a current interest.

Introduction

Since the early 1980's when Albert Tarantola and Patrick Lailly introduced their groundbreaking work by solving the waveform inversion problem as a sequence of pre-stack migrations (Lailly, 1984; Tarantola 1984), there has been a sense of excitement about the prospect. This workshop defines "Full waveform inversion" as "any automatic imaging processing which aims to exploit the full information contained in the seismic data …". The hope has always been that we can ultimately squeeze all the available information content out of the seismic waveform, leaving no wiggle unused in our quest.

The practice of waveform inversion has however also long been recognized as a technique loaded with a great many practical difficulties. In the 1980's and 1990's computational resources were the main limitation that prevented extensive testing. These limitations have been overcome, and researchers can now realistically consider even 3D waveform inversion. However, the convergence of descent-based methods continues to be a major concern. Two main convergence issues have been clear for some time:

1. Waveform misfit is a notoriously ill-behaved measure – only at low frequencies or with very accurate starting models can we avoid convergence to (false) local minima.
2. Even if we overcome the local minimum problem, the waveform misfit of small-offset reflection events is insensitive to the velocity model, and moreover suffers from a fundamental tradeoff between depth and velocity.

These considerations led us to make several key strategic decisions quite early:

1. We implementation the forward and inverse problem entirely in the frequency-domain. Most importantly, this decouples the high frequencies from the early stages of the inversion, thus partly mitigating the problem of local minima.
2. We focussed on non-standard seismic configurations, especially the crosswell and wide-angle (refraction) geometries which are sensitive to velocity models.
3. We emphasized in all our work the critical importance of low frequency data.

These decisions put us in some cases in opposition to the mainstream of exploration research and development, but they played a key role in our success in a number of blind tests, and in a range of interesting data problems. In order to distinguish our approach, we have usually referred to this as "Waveform Tomography", as the crosswell and refraction configurations are amenable to tomographic reconstructions of the velocity structure.

Convergence, starting models, frequencies, and propagation distances

A very useful criteria for convergence can be obtained by simple considerations of traveltime errors, and propagation distances measured in wavelengths: For a descent-based method to converge, the traveltime error for a given event has to be less than a half-wavelength (Sirgue, 2004). For a traveltime error of δt this leads to the inequality $\delta t < \lambda / 2c$. Relative to the total arrival time of the event, T, therefore

$$\frac{\delta t}{T} < \frac{\lambda}{2cT} \quad or \quad \frac{\delta t}{T} < \frac{1}{2N_\lambda}, \tag{1}$$

where N_λ is the propagation distance in wavelengths. As an example of the utility of equation (1), we can estimate that for a propagation distance of 50 wavelengths from source to receiver, the relative traveltime errors in the starting model must be of the order of $1/(2 \cdot 50)$= 1% or less, a very stringent requirement not often met in practice. Clearly we can reduce the stringency of this requirement by reducing the number of wavelengths between source and receiver, which in turn requires either lower frequencies, or reducing the offset.

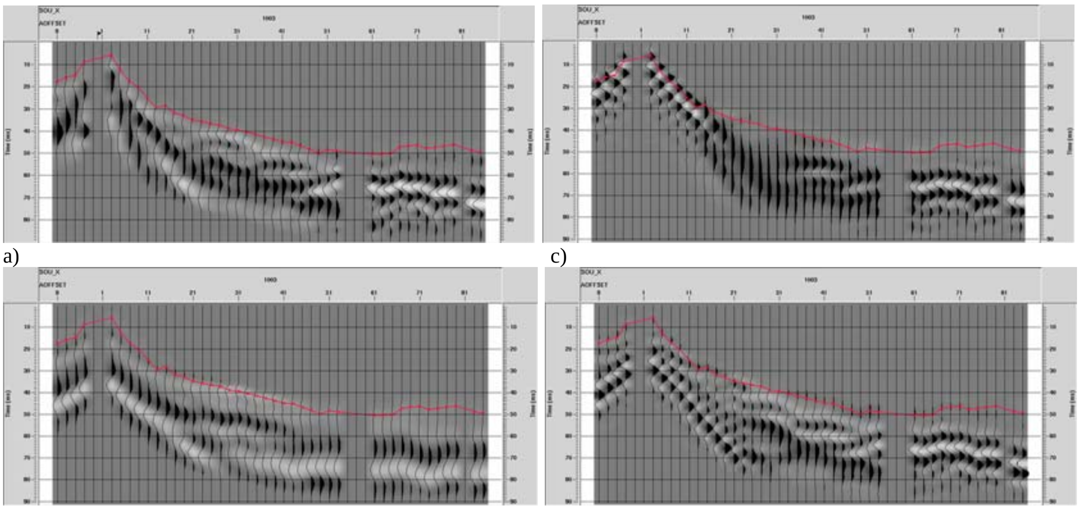
Two paradoxes arise from these considerations:

1. Large offsets in the data are good, as they are required to constrain the low wavenumbers in the velocity model. However, large offsets are also bad, as too many wavelengths lead to large values of N_λ, and hence to convergence problems.
2. Time domain inversions are good, as they contain more frequency components, thus larger signal-to-noise ratios, and they allow localization of events in time. However, time domain inversions are also bad, as the inclusion of high frequencies at the early stages leads again to large values of N_λ, and hence again to convergence problems.

These two paradoxes govern much of our current state-of-the-art in waveform inversion, and can lead to two very common “inverse crimes”: For synthetic tests, it is very tempting to either reduce δt (by artificially improving the accuracy of the starting model), or to reduce N_λ (by either reducing the offsets, or reducing the starting frequencies). Either of these approaches can lead to artificially optimistic results, and it should always be remembered that part of the value of synthetic tests is to generate results that illustrate not only potential benefits, but potential pitfalls.

Data fit – two cautionary tales

While synthetic tests and benchmark tests offer the value of allowing evaluation against a known model, real data do not always afford ground truth comparisons. Very often the only evaluation we can make is based on the goodness of fit of the data. This can be a very subjective criterion, as we illustrate in Figure 1: The panels depict a real shot gather from a shallow refraction survey (Smithyman and Pratt, 2008), and three synthetic shot gathers from the “best” models at various stages of the research project. Although at each stage it was felt the goodness of fit justified confidence in the models, the models continued to improve as the inversion parameters were adjusted. The models at each stage were quite different, leading to the conclusion that very high standards of waveform goodness of fit are required before a given model should be accepted.

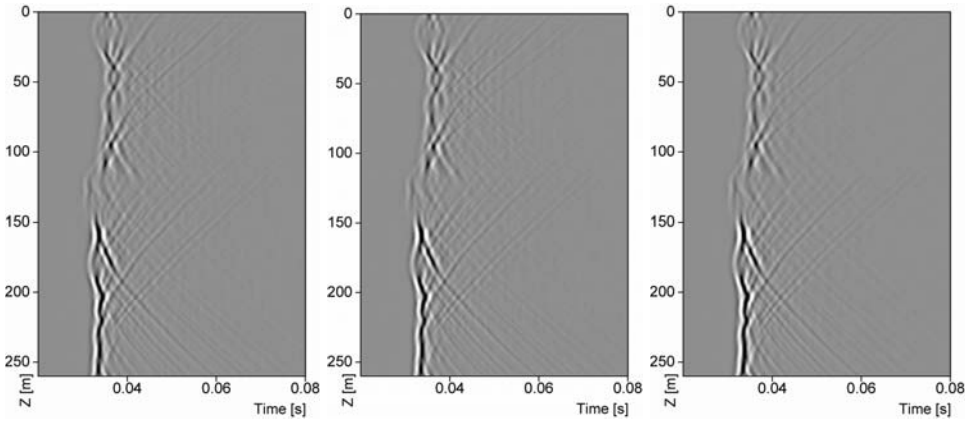


a) c)

d) e)

Figure 1 a) Real shot gather data, following edits and preprocessing. b) – d) Predicted shot gather data in our “best” models at various stages during the research project, using a variety of preprocessing approaches. At each stage we felt we had actually solved the problem.

A second example of the critical importance of the data fit is provided in Figure 2, which compares “true” data with two predicted datasets in a synthetic test of crosswell waveform tomography. The two predicted datasets both present a visually impressive goodness of fit, yet the models differ quite radically: In the first model the geological target is nonattenuating, while in the second example the target has a heterogeneous seismic-Q structure (Kamei and Pratt, 2008).

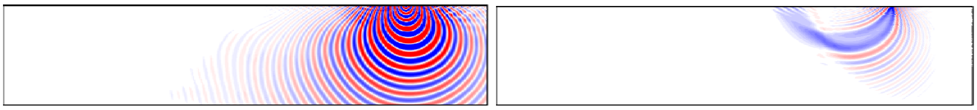


a) “True” crosswell data b) Synthetic data in infinite-Q c) Synthetic data in variable Q

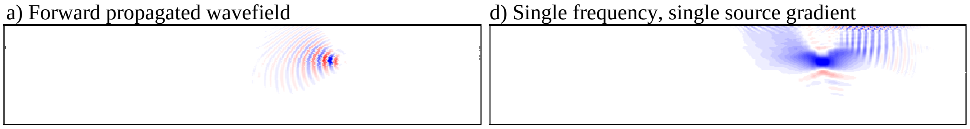
Figure 2 In a) synthetic data have been generated in a complex model, containing highly heterogeneous structures in both the seismic velocity, and in seismic-Q structure. In b) the predicted data are shown for an infinite-Q model inverted from the data in a), and in c) predicted data are shown for a variable-Q model inverted from the same data.

Wave propagation, gradient computation and quality control in the frequency domain

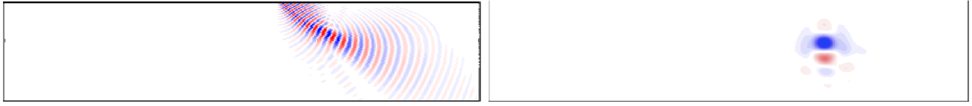
The examples above show that very high fidelity should be expected in the goodness-of-fit between field data and predicted synthetic data. Figures 1 and 2 illustrated goodness-of-fit using conventional time-domain data displays. It has been our experience that the goodness-of-fit can usually be anticipated by careful examination of the frequency-domain displays generated at the intermediate stages of waveform tomography. Figure 3 depicts several of the wavefields that are typically generated during a single-frequency stage of waveform tomography, while Figure 4 depicts the frequency-domain data displays used to evaluate the progression of the goodness-of-fit of the waveform tomography models.



a) Forward propagated wavefield d) Single frequency, single source gradient



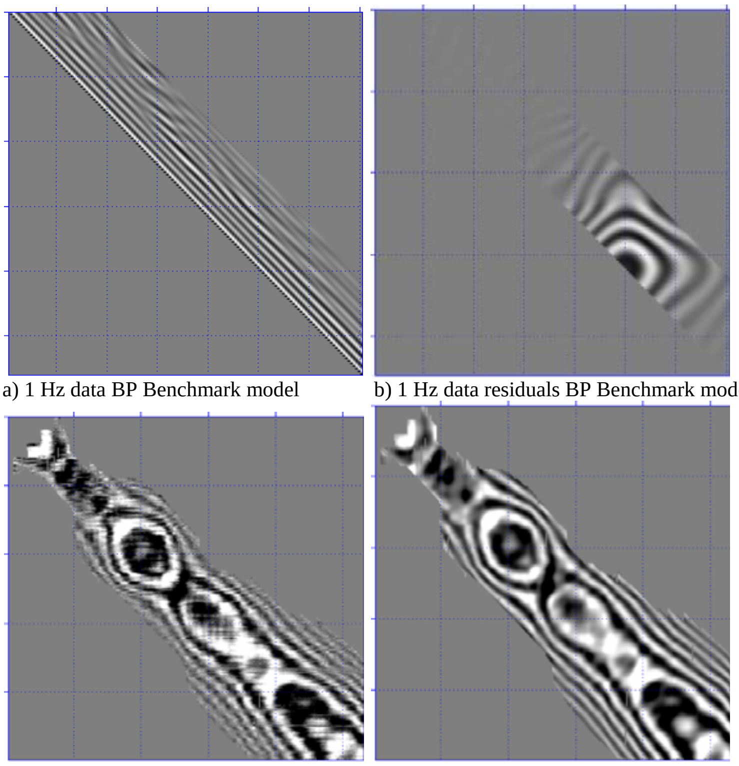
b) Wavefield differences (due to perturbation) e) Single frequency, multiple source gradient



f) Backpropagated differences g) Iterative waveform tomography result

Figure 3 Frequency-domain waveform inversion. The illustrations show 3 Hz single frequency results in a smooth version of the 2004 BP Velocity Benchmark model, with the inclusion of a small test perturbation. As in the original benchmark, we used a 15 km streamer. Panel b) can only be computed with a knowledge of the test perturbation, but panels a) and c) through f) can be computed from the data in the starting model.

The wavefields depicted in Figure 3 are recorded at streamer locations, with one such set of numbers for each source location. Such a dataset can be displayed as a frequency-domain data map, such as those shown in Figure 4, for the BP Benchmark perturbation study of Figure 3, and also for a crosswell dataset provided by BP North America Gas (Pratt, Sirgue, Hornby and Wolfe, 2008). Evaluation of waveform fit can readily be assessed in these displays.



a) 1 Hz data BP Benchmark model b) 1 Hz data residuals BP Benchmark model

c) 500 Hz data, BP Crosswell data d) 500 Hz predicted data, BP Crosswell data

Figure 4 Frequency domain data panels. a) and b) are model study data, based on the BP Benchmark Velocity model. c) and d) are real data and predicted data from a BP Crosswell dataset.

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